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Two interacting particles in a disordered chain II: Critical statistics and maximum mixing of the one body states

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Abstract. For two particles in a disordered chain of length L with on-site interaction U, a duality transformation maps the behavior at weak interaction onto the behavior at strong interaction. Around the fixed point of this transformation, the interaction yields a maximum mixing of the one body states. When $L \approx L_1$ (the one particle localization length), this mixing results in weak chaos accompanied by multifractal wave functions and critical spectral statistics, as in the one particle problem at the mobility edge or in certain pseudo-integrable billiards. In one dimension, a local interaction can only yield this weak chaos but can never drive the two particle system to full chaos with Wigner-Dyson statistics.

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The competition between two body (electron-electron) interaction and one body kinetic energy in disordered systems is a fundamental problem of permanent interest. We denote by U, t and W the parameters characterizing the interaction, the one body kinetic energy and the fluctuations of the random potential for a d-dimensional system of size L. When U is small, the N-body eigenstates are close to the symmetrized products of one body states (Slater determinants for spinless fermions) which contain the effects of t and W completely. The effect of U can be treated as a perturbation, yielding a mixing of those symmetrized products. When U increases, the consequence of this mixing is that an increasing number of one body states is needed to describe the exact N-body states. If the one body states are localized by the disorder, delocalization in real space results from this mixing. This is why the interaction can induce in certain cases metallic behavior in a system which would be an insulator otherwise. When U is large and dominates, one can get on the contrary a correlated insulator which might be metallic at weaker interaction. A Wigner crystal pinned by disorder is a good example of such an interaction-induced insulator. In the large-U limit, t becomes the small parameter, and one expands in powers of t^2/U. The issue is to know the range of validity of these perturbative approaches, and to describe how the system goes from the first limit to the second when U increases. For this purpose, we consider a one-dimensional disordered lattice with on-site interactions.

We discuss the simple case of two electrons with opposite spins (the orbital part of the wavefunction is symmetric as in the case of two bosons). For the main sub-band of states centered around $E = 0$, both in the limits where $U = 0$ (free bosons) and $U = \infty$ (hard-core bosons), the two body states can be described in terms of two one body states. We use a duality transformation $U \rightarrow at^2/U$ to map the small U-limit onto the large U-limit ($a \approx \sqrt{24}$). We first prove that the lifetimes of the free boson states and of the hard-core boson states are equal at the fixed point U_c of the duality transformation. At U_c one has the maximum mixing of the one body states by the interaction and the enhancement factor [1] is maximum for the two particle localization length L_2. Far from U_c, L_2 is smaller and satisfies the duality relation. $L_2 \rightarrow L_1$ (the one particle localization length) both for $U \rightarrow 0$ and $U \rightarrow \infty$. The study of the signature of this duality transformation on the spectral fluctuations is very interesting. For $E = 0$, taking $L = L_1$ and increasing U, one gets two thresholds defining a range of interaction $U_F \leq U \leq U_H$. Outside this range, the levels are almost uncorrelated. Inside this range, the level repulsion is maximum, but does not reach the universal Wigner-Dyson (W-D) repulsion. The two particle system is not fully chaotic, but exhibits a weak chaos which is not arbitrarily situated between Poisson (integrable) and Wigner (chaos). The spacing distribution $p(s)$ between consecutive energy levels and the statistics $\Sigma_2(E)$ (variance of the number of energy levels inside an energy interval E) are characteristic of the third known universality class [2]. One finds $p(s) \approx 4s\exp(-2s)$

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and $\Sigma_2(E) \approx 0.16 + 0.41E$ for periodic boundary conditions. This is very close, if not identical, to the distributions found in many "critical" one body systems, such as an electron in a 3d random potential at the mobility edge [2–4] or in certain pseudo-integrable quantum billiards (rational triangles [5], rough billiards [6] and Kepler problem [7]). Furthermore, $p(s)$ saturates to $4s\exp(-2s)$ for $U \approx U_c$ only when the ratio $1 \leq L_1/L \leq 10$. We conclude that a local interaction can never drive the two particle system to full quantum chaos with Wigner-Dyson statistics in one dimension, but can at most yield weak critical chaos in a certain domain of interaction and of the ratios L_1/L. We show in addition that this weak chaos is accompanied by multifractal wavefunctions, in agreement with reference [8].

Each one particle Hamiltonian is given by

$$H_0 = t\sum_{n=1}^{L-1}(|n\rangle\langle n+1| + |n+1\rangle\langle n|) + \sum_{n=1}^{L} V_n|n\rangle\langle n| \quad (1)$$

where V_n is uniformly distributed inside $[-W, +W]$ and the interaction is described by

$$H_{int} = U\sum_{n=1}^{L}|nn\rangle\langle nn|. \quad (2)$$

Denoting $|\alpha\rangle$ the one body state of energy ϵ_α and the amplitude $\langle n|\alpha\rangle = \Psi_\alpha(n)$, only two one body states $|\alpha\rangle$ and $|\beta\rangle$ are necessary to describe a free boson state $|fb\rangle = |\alpha\beta\rangle$ with components $\langle n_1 n_2|fb\rangle$ given by

$$(\Psi_\alpha(n_2)\Psi_\beta(n_1) + \Psi_\alpha(n_1)\Psi_\beta(n_2))/\sqrt{2}. \quad (3)$$

In this free boson basis, the interaction matrix elements $\langle\alpha\beta|H_{int}|\gamma\delta\rangle = 2UQ^{\gamma\delta}_{\alpha\beta}$ where

$$Q^{\gamma\delta}_{\alpha\beta} = \sum_{n=1}^{L}\Psi_\alpha(n)\Psi_\beta(n)\Psi_\gamma(n)\Psi_\delta(n). \quad (4)$$

This Q-matrix has been studied [8] in one dimension for $L \geq L_1$. It was found that its lines, where are the terms $Q^{\gamma\delta}_{\alpha\beta}$ coupling a given $|\alpha\beta\rangle$ to all the other $|\gamma\delta\rangle$, is a multifractal measure in the $|\gamma\delta\rangle$ space. Therefore the effective density ρ_2^{eff} of $|fb\rangle$ states coupled by the square of the interaction matrix elements is reduced. For $L \approx L_1$, one has $\rho_2^{eff}(L_1) \propto L_1^{1.75}$, instead of the total two body density $\rho_2 \propto L_1^2$. This was confirmed by a study of their Fermi golden rule decay.

As noticed in reference [9], there is a useful representation for an on-site interaction $U \to \pm\infty$, composed by a small set of $L_\mathrm{M} = L$ "molecular" states $|nn\rangle$ and by a large set of $L_\mathrm{H} = L(L-1)/2$ hard core boson states $|hc\rangle$ built from re-symmetrized Slater determinants. Their components $\langle n_1 n_2|hc\rangle$ are given by

$$\frac{(\Psi_\alpha(n_2)\Psi_\beta(n_1) - \Psi_\alpha(n_1)\Psi_\beta(n_2))\mathrm{sgn}(n_2 - n_1)}{\sqrt{2}}\,. \quad (5)$$

The re-symmetrization is insured by the function $\mathrm{sgn}(x) := x/|x|$. For hard wall boundaries (which we assume in our discussion), one has to use the same one body states $|\alpha\rangle$ and $|\beta\rangle$ both for $|fb\rangle$ and $|hc\rangle$. However, if the system is closed on itself and forms a ring threaded by a flux Φ, the re-symmetrization on a torus reveals interesting topological aspects (see Ref. [10]). One has to use different one body states in the two limits which differ in Φ by half a flux quantum (to periodic boundary conditions for $U = 0$ correspond anti-periodic boundary conditions for $U \to \pm\infty$).

In this basis, the two body Hamiltonian $\mathcal{H}$ has the structure

$$\mathcal{H} = \begin{bmatrix} H_M & H_C \\ H_C^T & H_H \end{bmatrix}. \quad (6)$$

H_M and H_H are $L_M \times L_M$ and $L_H \times L_H$ diagonal matrices with entries $U + 2V_n$ and $\epsilon_{hc} = \epsilon_\alpha + \epsilon_\beta$, respectively. H_M and H_H are coupled by a rectangular matrix H_C resulting from the kinetic terms $\sqrt{2}\,t\sum_{n=1}^{L-1}(|n,n+1\rangle\langle n,n| + |n,n\rangle\langle n,n+1| + \text{h.c.})$ of $\mathcal{H}$. The matrix elements of H_C are equal to $\sqrt{2}\,t\left(\Psi_\beta(n)\hat{D}\Psi_\alpha(n) - \Psi_\alpha(n)\hat{D}\Psi_\beta(n)\right)$, where $\hat{D}\Psi_\alpha(n) := \Psi_\alpha(n+1) - \Psi_\alpha(n-1)$. The states $|hc\rangle$ of energy $\epsilon_{hc} \approx 0$ are coupled by a term of order t to the few states $|nn\rangle$ of energy $\approx U$. Their lifetime becomes infinite when $U \to \pm\infty$.

Projecting an eigenstate $|A\rangle$ of energy E_A onto the states $|nn\rangle$ and $|hc\rangle$

$$|A\rangle = \sum_{n=1}^{L_M} c_A^n|nn\rangle + \sum_{hc=1}^{L_H} c_A^{hc}|hc\rangle\,, \quad (7)$$

one finds the relation

$$\begin{bmatrix} H_M + J_M(E_A) & 0 \\ 0 & H_H + J_H(E_A) \end{bmatrix}\begin{bmatrix} C_A^M \\ C_A^H \end{bmatrix} = E_A\begin{bmatrix} C_A^M \\ C_A^H \end{bmatrix},$$

which holds for arbitrary U. C_A^M and C_A^H are vectors of L_M components c_A^n and L_H components c_A^{hc} respectively. The matrix $J_M(E_A)$ has L_M^2 elements given by

$$J_M(E_A, nn, mm) = \sum_{hc=1}^{L_H}\frac{(\langle nn|H_C|hc\rangle\langle hc|H_C|mm\rangle)}{E_A - \epsilon_{hc}} \quad (8)$$

and the matrix $J_H(E_A)$ has L_H^2 elements of the form

$$J_H(E_A, hc, \tilde{hc}) = \sum_{n=1}^{L_M}\frac{\langle hc|H_C|nn\rangle\langle nn|H_C|\tilde{hc}\rangle}{U + 2V_n - E_A}\,. \quad (9)$$

We consider the main sub-band of states with energy $E_A \approx 0$, resulting from the mixing by a perturbation of order t^2/U of a few states $|hc\rangle$ for which $J_H(E_A)$ can be simplified. Assuming $W, t \ll U$ and $U + 2V_n - E_A \approx U$, one has to evaluate $\sum_{n=1}^{L}\langle hc|H_C|nn\rangle\langle nn|H_C|\tilde{hc}\rangle$. This expression is composed of 12 sums over n, each of them having the form

$$\tilde{Q}^{\gamma\delta}_{\alpha\beta} = \sum_{n=1}^{L}\Psi_\alpha(n)\Psi_\beta(n')\Psi_\gamma(n)\Psi_\delta(n'') \quad (10)$$

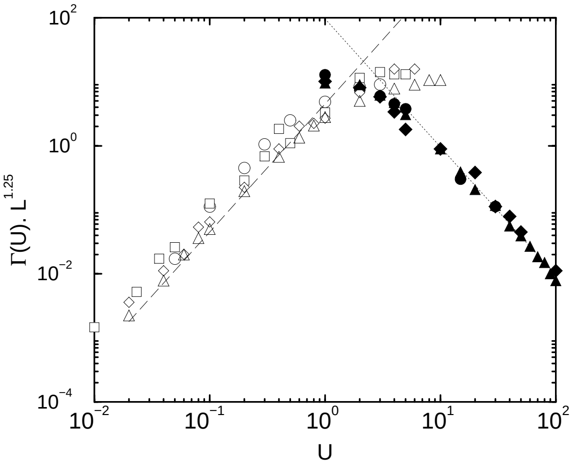

Fig. 1. Γ_0 (open symbols) and Γ_∞ (filled symbols) for $L = 25$ (triangles), $L = 76$ (diamonds), $L = 150$ (squares) and $L = 200$ (circles). Dashed (dotted) lines are the Fermi golden rule expression of equation (13) (Eq. (14)).

with various combinations of $n', n'' = n \pm 1$. Therefore $\tilde{Q}^{\gamma\delta}_{\alpha\beta}$ is not exactly equal to the $Q^{\gamma\delta}_{\alpha\beta}$ occuring around the free boson limit. However, this difference should not be statistically relevant and if one neglects it, one finds

$$J_H(E_A, hc, \tilde{hc}) \approx \pm 2(24)^{1/2}\frac{t^2}{U}Q^{\gamma\delta}_{\alpha\beta}. \tag{11}$$

This establishes the duality transformation $U \rightarrow \sqrt{24}(t^2/U)$ for $E \approx 0$ which maps the distribution of the coupling terms between the $|fb\rangle$ when U is small onto the distribution of the coupling terms between the $|hc\rangle$ when U is large. To illustrate this duality around the fixed point $U_\mathrm{c} = (24)^{1/4}t$, we have numerically calculated the average over the disorder of the local density of states

$$\rho_A(E) = \sum_{\alpha\beta} |c^{\alpha\beta}_A|^2 \delta(E + E_A - \epsilon_\alpha - \epsilon_\beta) \tag{12}$$

for $L = L_1$ and $E_A \approx 0$. We have observed that $\langle\rho_A(E)\rangle$ can be fitted by a Lorentzian curve of width Γ_0 (if $\alpha\beta$ denotes the states $|fb\rangle$) or Γ_∞ (if $\alpha\beta$ denotes the states $|hc\rangle$). Fermi's golden rule gives for the widths Γ at $L \approx L_1$

$$\Gamma_0 \approx 2\pi U^2 \frac{1}{L_1^3}\rho_2^{eff}(L_1) \tag{13}$$

$$\Gamma_\infty \approx 2\pi \frac{24t^4}{U^2}\frac{1}{L_1^3}\rho_2^{eff}(L_1) \tag{14}$$

where $\rho_2^{eff} \approx L_1^{1.75}/t \neq \rho_2(L_1) \approx L_1^2/t$ because of the multifractality of the Q-matrix. As shown in Figure 1, the widths Γ obey the duality property $\Gamma_0(U) = \Gamma_\infty(\sqrt{24}t^2/U)$, and are described by the above golden rule approximations. When $U \approx U_\mathrm{c}$, the lifetime of the free boson states is equal to the one of the hard core boson states.

When U increases, the statistical ensemble associated to the two particle system exhibits a crossover from one preferential basis (the free boson basis) to another preferential basis (the hard core boson basis). At U_c, the statistical ensemble is in the middle between the two preferential bases, the mixing of the one body states is maximum, and the localization length L_2 therefore is maximum. When $E = 0$ and U varies, a transition occurs in the two body spectrum in 1d which is somewhat reminiscent of the one body case in 3d when $E = 0$ and W varies. In the two cases, one can expand in powers of U/t or t/U, of W/t or t/W when the system is under or above the critical threshold, respectively. The analogy is of course not complete, since the two body spectrum in 1d has essentially Poisson statistics in the two perturbative limits, while the one body spectrum in 3d exhibits W-D statistics for $W \ll t$ and Poisson statistics when $W \gg t$. Nevertheless, the question whether or not the spectral fluctuations are of the same kind in the vicinity of the threshold deserves to be investigated.

Before showing the results, two arguments can be mentioned: (i) A multifractal Q-matrix is incompatible with Wigner-Dyson level repulsion in 1d. The states $|fb\rangle$ or $|hc\rangle$ are directly coupled by U or t^2/U to an effective density $\rho_2^{eff} < \rho_2$. Therefore, nearest neighbors in energy are very likely uncorrelated, enhancing the probability to find level spacings small compared to their average. (ii) Looking at equation (2), one may assume that a broad distribution of the matrix elements of H_M may yield a single dominant coupling term. This allows us to consider that the states $|hc\rangle$ are mainly coupled *via* a few states $|nn\rangle$. This is not far from the case discussed by Bohr and Mottelson [11] (coupling via a single state) where the consecutive levels E_A of $\mathcal{H}$ satisfy the conditions $\epsilon_{hc} < E_A < \epsilon_{hc+1} < E_{A+1}$, ϵ_{hc} being the consecutive levels of H_H. Since the statistics of H_H is essentially Poissonian, this forbids to have the Wigner-Dyson rigidity for the E_A. The most rigid spectrum would be achieved by putting the E_A exactly in the middle of two consecutive ϵ_{hc}. It is straightforward to find that $p(s)$ would then be equal to $p_\mathrm{c}(s) = 4s\exp(-2s)$. This "semi-Poisson" distribution, where $p_\mathrm{c}(s) \propto s$ for $s \ll 1$ (as the Wigner surmise $p_\mathrm{W}(s) = (\pi/2)s\exp(-\pi s^2/4)$), and which decays as $\exp(-s)$ for $s \gg 1$ (as the Poisson distribution $p_\mathrm{P}(s)$), is attracting increasing interest since it characterizes [5] weak chaos in systems which are "critical" (*i.e.* which can be mapped [7] in a certain "integrable" basis onto an Anderson model at its critical point). It was shown in reference [12] that a plasma model for the energy levels can reproduce the level statistics of a disordered conductor if the logarithmic Wigner-Dyson level repulsion was screened at an energy scale given by the Thouless energy E_T. Since E_T is of order of the mean level spacing Δ_1 at the mobility edge, one can expect that a short range plasma model should also give this "semi-Poisson" distribution. This was recently confirmed [5] for a logarithmic level repulsion truncated to the first consecutive level, where one finds $p(s) = 4s\exp(-2s)$ and $\Sigma_2(E) = 1/16 + E/2$. After averaging over certain boundary conditions, $p_\mathrm{c}(s)$ was also found to give [4] a good fit for the $p(s)$ of the one body spectrum in 3d at the mobility edge.

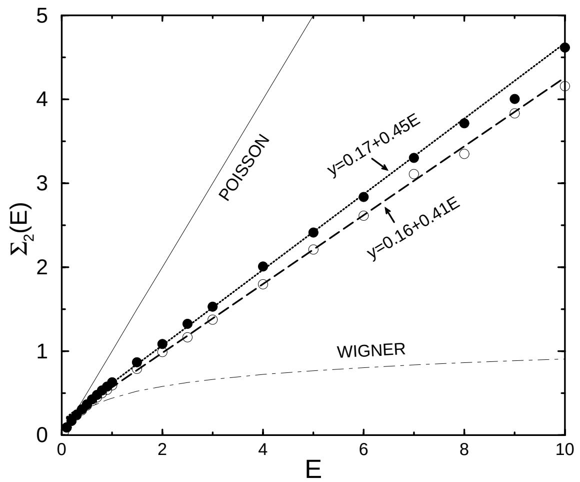

Fig. 2. $\Sigma_2(E)$ for $L = L_1 = 100$ and $U = 1.25\ E$ in units of the mean level spacing. Full (empty) circles correspond to hardwall (periodic) boundary conditions.

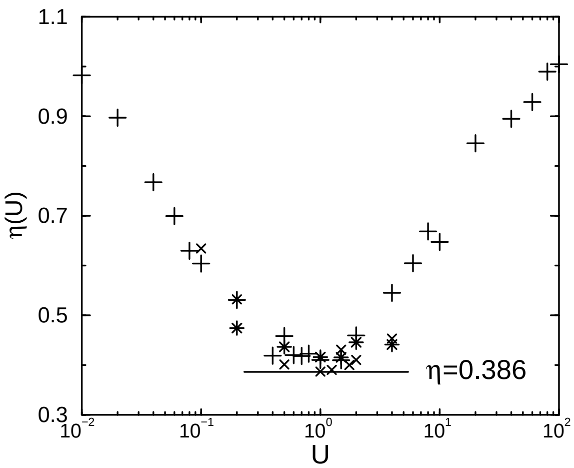

Fig. 3. Weak chaos and interaction for $L = L_1$: Crosses, pluses and stars represent $\eta(U)$ for $L = 100$, $L = 150$ and $L = 200$, respectively.

These observations lead us to study $p(s)$ as a function of U and of the ratio L_1/L around $E = 0$, where ρ_2 has a van Hove singularity for $W = U = 0$. The disorder and the interaction remove the singularity and ρ_2 develops a small plateau around $E \approx 0$. It is inside this plateau that $p(s)$ and Σ_2 have been investigated. For $U = 1$ and $L = L_1 = 150$, the spacing distribution $p(s)$ is shown in Figure 4, in good agreement with the "semi-Poisson" distribution. In the inset, one can see that this is specific to the two particle problem, and does not characterize the single particle spectrum for the same chain of size L_1.

The statistics $\Sigma_2(E)$ displays also the behavior that one expects at a critical point or for a short range plasma model. $\Sigma_2(E) \approx \chi_0 + \chi_1 E$ is shown in Figure 2. The values of the compressibility $\chi_1 = \lim_{E\to\infty} \Sigma_2(E)/E$ seem to depend on the boundary conditions for the first consecutive levels, as for the Anderson transition in 3d. The values for the two particle system 0.41 (periodic) and 0.45 (hard wall) coincide with the values given [4] by Braun *et al.* for certain combinations of boundary conditions in the different directions. This sensitivity of χ_1 to the boundary conditions may disappear for larger E (smaller time scales), as it is the case for the one particle spectrum at the mobility edge [13].

We now study the domain of validity for weak chaos and universal critical statistics. To measure the deviation of $p(s)$ from the $P_P(s)$ or $P_W(s)$, we use the functional

$$\eta(U) = \frac{\int_0^b ds(p(s) - p_W(s))}{\int_0^b ds(p_P(s) - p_W(s))} \qquad (15)$$

with $b = 0.4729$. A Poisson spectrum gives $\eta = 1$, a Wigner-Dyson spectrum gives $\eta = 0$ and "semi-Poisson" corresponds to $\eta_c = 0.386$.

First, we vary U, imposing the relation $L = L_1$. We assume L_1 to be given by the weak disorder formula $L_1 \approx 25/W^2$. In the limits $U \to 0$ and $U \to \infty$, the TIP-levels are given by $\epsilon_{bf} = \epsilon_{hc} = \epsilon_\alpha + \epsilon_\beta$ which are uncorrelated (at least on energy scales smaller than the one particle level spacing Δ_1). This gives $p_P(s)$ for the TIP-spectrum. One can see in Figure 3 that $p(s)$ deviates for small U and t^2/U from $p_P(s)$ observed at $U = 0$ and $U = \infty$. $\eta(U)$ exhibits a plateau ($U_{\rm F} < U < U_{\rm H}$) at the value $\eta_c \approx 0.386$ which characterizes $p_c(s)$). We suggest that U_F and U_H are given by the conditions which hold [14] for systems in which there is no coupling term between consecutive energy levels. In agreement with the general picture developed in reference [14] (see also Ref. [15]), the threshold appears when the strength of the coupling terms becomes of the order of the spacing $1/\rho_2^{eff}$ of the directly coupled levels. This gives $2U_F/L_1^{3/2} \approx 4t/L_1^{1.75}$ and $2\sqrt{24}t^2/(U_H L_1^{3/2}) \approx 4t/L_1^{1.75}$, respectively, and seems to account for the size of the plateau of the curve $\eta(U)$. This favours a line of critical points rather than an isolated point when the condition $L = L_1$ is imposed. The U-dependence of χ_1 exhibits the same plateau as $\eta(U)$. The results shown in Figure 2 for $U = 1.25$ do not vary inside the plateau.

Second, for $U = 1.25$ and $L = 100$, we vary the disorder parameter W in order to study the role of the ratio L_1/L. When $L \gg L_1$, the small fraction of TIP-states reorganized by U disappears behind the set of TIP-states which are not re-organized by U and which remain uncorrelated. $p(s)$ does not differ from $p_P(s)$. On the contrary, when $L \ll L_1$, we approach the clean limit which can be easily understood: For $W = 0$, the total TIP-momentum K is conserved and the Hamiltonian matrix has a block diagonal form in the free boson eigenbasis. Denoting by $\epsilon_\gamma = 2\cos k_\gamma$ the eigenenergy of a single particle with momentum k_γ, and assuming periodic boundary conditions, one can easily show that the $(L + 1)/2$ TIP eigenenergies $E_A(K)$ of total momentum K modulo (2π) satisfy

$$\frac{2U}{L} \sum_{k_\gamma, k_\delta} \frac{1}{E_A(K) - \epsilon_\gamma - \epsilon_\delta} = 1. \qquad (16)$$

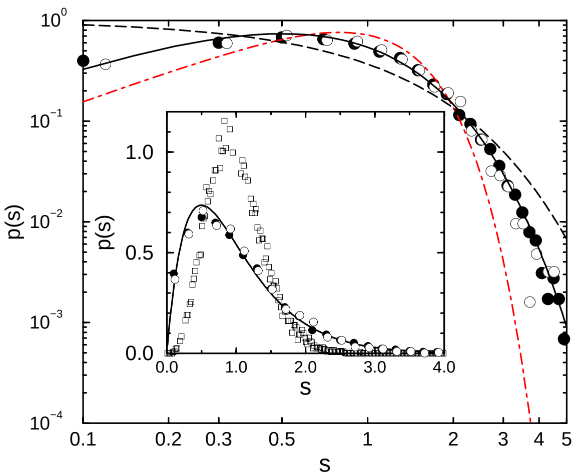

Fig. 4. TIP-$p(s)$ for $L = L_1 = 150$ and $U = 1$: Continuous, dotted and dashed lines correspond to the "Semi-Poisson", Wigner and Poisson distributions respectively. Full (empty) circles correspond to hardwall (periodic) boundary conditions. Inset: TIP-$p(s)$ (circles $U = 1$) compared to the one particle $p(s)$ (squares) for $L = L_1 = 150$.

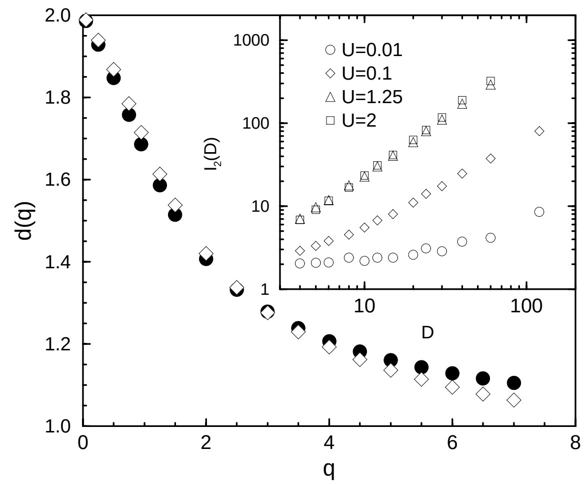

Fig. 6. $d(q)$ at $U = 2$ for $L = L_1 = 120$ in the bases $|fb\rangle$ (open symbols) and $|hc\rangle$ (filled symbols). The inset shows the establishment of the power-law behaviour of $I_2(D)$ for increasing values of U in the $|hc\rangle$ basis.

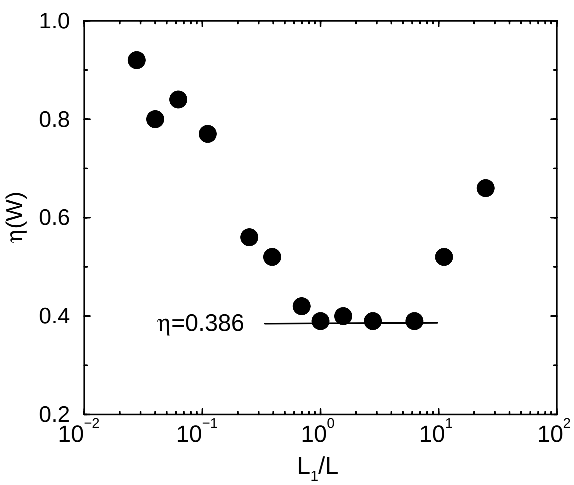

Fig. 5. Weak chaos and disorder for $U = 1.25$: $\eta(W)$ as a function of L_1/L with $L = 100$.

The summation is restricted to the momenta $k_\gamma + k_\delta = K$ modulo (2π). This sequence of $(L+1)/2$ levels exhibits the same property than in the case discussed by Bohr and Mottelson: For a given K, the levels for $U \neq 0$ alternate with the levels for $U = 0$. This sequence of $E_A(K)$ can only exhibit level repulsion between next nearest neighbors, but not the long-range repulsion necessary for Wigner-Dyson statistics. Since the total spectrum is the uncorrelated superposition of the L different sequences associated to different total momenta K, one gets for the TIP spectrum uncorrelated levels for any value of U when $W = 0$.

Between the two limits $L \ll L_1$ and $L \gg L_1$, one can see in Figure 5 a plateau where the spectrum is more rigid, $\eta(L_1/L)$ saturating to the "semi-Poisson" value. This plateau for $U = 1.25$ is observed for $1 \leq L_1/L \leq 10$.

A critical statistics for the spectrum is usually related to multifractal wave functions. We have studied the projection $C_A^{\alpha\beta}$ of an eigenstate $|A\rangle$ onto the states $|fb\rangle$ and onto the states $|hc\rangle$. Since the perturbation matrix elements (divided by U or t^2/U) are multifractal, one expects that this will eventually yield a multifractal structure for $|A\rangle$ in the two bases for large and small enough U, respectively. We have done the same analysis as in reference [8] to which we refer for technical details and references. We divide the plane (α, β) into $(L/D)^2$ boxes of size D and compute for each box i the probability $p_i = \sum_{(\alpha\beta)\in box_i} |C_A^{\alpha\beta}|$. After ensemble averaging, the moments $I_q(D) = \sum_i p_i^q$ have a power-law behaviour if U is large enough for the $|fb\rangle$ basis (inset of Fig. 6), and if U is small enough for the $|hc\rangle$ basis. So one finds $I_q(D) \propto D^{(q-1)d(q)}$, $d(q)$ defining the corresponding generalized Renyi dimensions. The $d(q)$ are roughly equal in the two bases (Fig. 6) and do not depend on U as soon as the power-law behaviour is established for $I_q(D)$, *i.e.* inside the line of fixed points.

In summary, we have shown that this simple two particle system in 1d is characterized by two perturbative regimes (in U or t^2/U). These regimes are dominated by the existence of a single preferential basis and are related to each other by a duality transformation. In the middle, the mixing of the one body states by the interaction is maximum, but the system is not chaotic (no Wigner-Dyson statistics). It exhibits the weak chaos of the critical systems, as confirmed by the spectral fluctuations. The curves $\eta(U,W)$, $\chi_1(U,W)$ and $d_q(U,W)$ in the $|fb\rangle$ and $|hc\rangle$ bases are consistent with the existence of a weak critical chaos for a given size in (U,W) domain located around U_c and $L_1/L \approx 1$. In one dimension, a local interaction can never drive a two particle system towards full chaos with Wigner-Dyson statistics.

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